# Theoretical study of isotope effects on the stereodynamics of $\mathbf{H}^{+}+\mathbf{H D}$ and its isotopic variant $\mathrm{D}^{+}+\mathbf{H D}$ 

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#### Abstract

The stereodynamics for $\mathrm{H}^{+}+\mathrm{HD}$ and its isotopic variant $\mathrm{D}^{+}+\mathrm{HD}$ were studied with a quasi-classical trajectory (QCT) method at a collision energy of 0.7 eV on the ground $1^{1} \mathrm{~A}^{\prime}$ potential energy surface (PES). The polarizationdependent differential cross-sections (PDDCSs) in the center-of-mass frame are presented here. Furthermore, the distribution of the angle between $\mathbf{k}$ and $\mathbf{j}^{\prime}, p\left(\theta_{r}\right)$ and the distribution of the dihedral angle $p\left(\phi_{r}\right)$ were calculated and are discussed. The results indicate that isotopic substitution exerts substantial effects on the differential cross-section and the product's rotational polarization.


Keywords Stereodynamics • Quasiclassical trajectory theory (QCT) method • Isotope effect

## Introduction

In interstellar chemistry, it is well known that a series of reactions are induced by the $\mathrm{H}_{3}{ }^{+}$ion, and that this subsequently leads to the formation of a great number of larger molecules [1, 2]. In addition, the $\mathrm{H}_{3}{ }^{+}$ion is the main component of hydrogen plasmas [1]. Since it is the typical ion-molecule reaction, the $\mathrm{H}_{3}{ }^{+}$system has always been used to test the quality of a new theory or experiment [2]. Due to its fundamental importance, the $\mathrm{H}^{+}+\mathrm{H}_{2}$ reaction and its isotopic variants have been investigated widely. Over the last decade, many theoretical and experimental studies have been carried out on this system [1-24]. In

[^0]2000 and 2006, two meetings entitled "Physics, Chemistry and Astronomy of $\mathrm{H}_{3}{ }^{+"}$ were held by the Royal Society (see papers from discussion meetings on the subject: [22, 23]). In these meetings, $\mathrm{H}_{3}{ }^{+}$was discussed from a variety of perspectives. On the other hand, previous studies on the Rydberg atom reaction $\mathrm{H}(\mathrm{n})+\mathrm{D}_{2} \rightarrow \mathrm{HD}+\mathrm{D}(\mathrm{n})$ using the Rydberg H -atom translation spectroscopy technique [13, 14] have shown quite similar results to those of the ionmolecule reaction $\mathrm{H}^{+}+\mathrm{D}_{2} \rightarrow \mathrm{HD}+\mathrm{D}^{+}$as calculated by the quasiclassical trajectory (QCT) method [21]. Thus, we can also use the ion plus diatom exchange process to investigate Rydberg atom plus diatom reaction dynamics. In order to fully understand the dynamics of an elementary reaction, it is necessary to study both the scalar and the vector properties of the reaction. In relation to chemical stereodynamics, Hsu et al. employed a new buffer field design in experimental measurements to deal with the orientation of the rotational angular momentum of a product molecule [25]. Aoiz et al. developed a complete classical treatment of the $\mathbf{k - j} \mathbf{j}^{\prime} \mathbf{k}^{\prime}$ vector correlation for atom-diatom reactions [26], which was applied in order to study the $\operatorname{Li}+\operatorname{HF}(v=1, j=1, m=0)$ reaction and other triatomic systems, and yielded results that were in good agreement with the experimental ones. Miranda and coworkers [24, 27] employed a quantum treatment to describe the stereodynamics of the $\mathrm{H}+\mathrm{D}_{2}$ reaction. They analyzed their calculated state-to-state results and found that the agreement between the quantum and quasiclassical calculations was quantitative for the polarization parameters. Based on density matrix techniques, angular momentum algebra, and their knowledge of the scattering matrix, they also presented a theoretical quantum method for describing the stereodynamics of four-atom reactions of the type $\mathrm{AB}+\mathrm{CD} \leftrightarrow \mathrm{ABC}+\mathrm{D}$, and applied their method to $\mathrm{H}_{2}+\mathrm{OH} \leftrightarrow \mathrm{H}_{2} \mathrm{O}+\mathrm{H}$ to reveal vector properties such as
the state-specific correlations between the polarizations of the diatomic molecules, their incident and recoil directions, etc. [28].
$\mathrm{H}^{+}+\mathrm{H}_{2}$ and its isotopic variants are governed by an insertion mechanism, and have been investigated by different methods. This reaction family usually has three product channels: reactive charge transfer (RCT), nonreactive charge transfer (NRCT) and reactive noncharge transfer (RNCT). Han and coworkers studied the $\mathrm{D}^{+}+\mathrm{H}_{2}$ and $\mathrm{H}^{+}+\mathrm{D}_{2}$ reactions, focusing on the nonadiabatic competition of these channels [15]. Their theoretical results agree with the experimental results quite well. Very recently, Han and coworkers also demonstrated that the three product channels present different energy-dependent isotope effects, with the most pronounced being predicted for the RNCT channel [16]. In addition, a great deal of study has focused on using statistical quantum, time-independent quantum mechanical, approximate quantum wavepacket, and quasicalassical trajectory approaches to investigate the $\mathrm{H}^{+}+\mathrm{H}_{2}$, $\mathrm{D}^{+}+\mathrm{H}_{2}$ and $\mathrm{H}^{+}+\mathrm{D}_{2}$ reactions [17-19]. For example, Jambrina et al. carried out WP-EQM (wavepacket exact QM), QCT and statistical quasi-classical trajectory (SQCT) calculations, and investigated the $\mathrm{H}^{+}+\mathrm{D}_{2}$ reaction for a range of collision energies, starting from the reaction threshold to 1.3 eV [2]. Song et al. studied the $\mathrm{H}^{+}+\mathrm{H}_{2}$ reaction with the QCT method and found that it is a complex-formation reaction [20]. All of these results indicate that the RNCT process is governed by an insertion mechanism.

Many theoretical studies of the $\mathrm{H}_{3}{ }^{+}$system have been carried out, but our understanding of this system is still incomplete. In order to gain more information about the isotope effect on this system, we carried out QCT calculations for the following reactions:

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\(\mathrm{H}^{+}+\mathrm{HD}(v=0, j=0) \rightarrow \mathrm{H}_{2}+\mathrm{D}^{+}(\)channel 1\() \rightarrow \mathrm{HD}+\mathrm{H}^{+}\)
(channel 2)
\(\mathrm{D}^{+}+\mathrm{HD}(v=0, j=0) \rightarrow \mathrm{DH}+\mathrm{D}^{+}(\)channel 1\() \rightarrow \mathrm{D}_{2}+\mathrm{H}^{+}\)
(channel 2)
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at a low collision energy of 0.7 eV , and we neglected all nonadiabatic processes.

## Computational methods

## Vector correlation and distribution

In Fig. 1, the center-of-mass (CM) frame is chosen to describe the correlations among $\mathbf{k}, \mathbf{k}^{\prime}$, and $\mathbf{j}^{\prime}$. The $x$-z plane is the so-called scattering plane containing the relative velocity vector $\mathbf{k}$ of the reagent and the relative velocity $\mathbf{k}^{\prime}$ of the product. The initial relative velocity vector $\mathbf{k}$ is along the $z$ axis. $\mathbf{j}^{\prime}$ is the rotational momentum of the product. $\theta_{t}$ is the


Fig. 1 The center-of-mass frame used to describe the correlations among $\mathbf{k}, \mathbf{k}^{\prime}$, and $\mathbf{j}^{\prime}$
angle between the relative velocity of the product and the relative velocity of the reagent, and is called the scattering angle. The angle $\theta_{r}$ is the so-called polar angle between the $z$ axis and the final rotational angular momentum. The azimuth angle $\varphi_{t}$ and $\theta_{r}$, together determine the direction of the final rotational angular momentum $\mathbf{j}^{\prime}$.

In the CM frame, the full three-dimensional angular distribution associated with $\mathbf{k}, \mathbf{k}^{\prime}$, and $\mathbf{j}^{\prime}$ is represented by a set of generalized polarization-dependent differential crosssections (PDDCSs) [29-32]. The fully correlated CM angular distribution is written as the sum [33]
$p\left(\omega_{t}, \omega_{r}\right)=\sum_{k q} \frac{[k]}{4 \pi} \frac{1}{\sigma} \frac{d \sigma_{k q}}{d \omega_{t}} C_{k q}\left(\theta_{r}, \phi_{r}\right)$,
where $\omega_{t}=\theta_{t}, \phi_{t}$ and $\omega_{r}=\theta_{r}, \phi_{r}$ refer to the coordinates of the unit vectors $\mathbf{k}^{\prime}$ and $\mathbf{j}^{\prime}$ along the directions of the product's relative velocity and angular momentum vectors in the CM frame, respectively [34]. [k]=2k+1, $(1 / \sigma)\left(d \sigma_{k q} / d \omega_{t}\right)$ is a generalized polarization-dependent differential crosssection (PDDCS), and $C_{k q}\left(\omega_{r}\right)$ are modified spherical harmonics. In our work, $\frac{2 \pi}{\sigma} \frac{d \sigma_{00}}{d \omega}, \frac{2 \pi}{\sigma} \frac{d \sigma_{20}}{d \omega}, \frac{2 \pi}{\sigma} \frac{d \sigma_{22+}}{d \omega}$ and $\frac{2 \pi}{\sigma} \times$ $\frac{d \sigma_{21-}}{d \omega}$ are calculated. PDDCSs are expanded up to $\mathbf{k}_{1}=7$, and they show good convergence.

Customarily, double-vector correlations ( $\mathbf{k}-\mathbf{j}^{\prime}, \mathbf{k}-\mathbf{k}^{\prime}$ or $\mathbf{k}^{\prime}$ $\mathbf{j}^{\prime}$ ) can be expanded as a Legendre series. Thus, $\mathbf{k}-\mathbf{j}^{\prime}$ is expanded as the distribution function $p\left(\theta_{r}\right)$ [33, 35-37]:
$p\left(\theta_{r}\right)=\frac{1}{2} \sum_{k}[k] a_{0}^{k} p_{k}\left(\cos \theta_{r}\right)$.
The $a_{0}^{k}$ coefficients are polarization parameters:
$a_{0}^{k}=\left\langle p_{k}\left(\cos \theta_{r}\right)\right\rangle$.
The angular brackets indicate an average over all reaction trajectories. When $p\left(\theta_{r}\right)$ is expanded up to $k=18$, it shows
good convergence. Triplet vector correlations ( $\mathbf{k}-\mathbf{k}^{\prime}-\mathbf{j}^{\prime}$ ) can be described by the dihedral angle $\boldsymbol{\phi}_{r}$ distribution. This distribution of $\phi_{r}$ is expanded as a Fourier series [33, 35-37]:
$p\left(\phi_{r}\right)=\frac{1}{2 \pi}\left(1+\sum_{n, \text { even } \geq 2} a_{n} \cos n \phi_{r}+\sum_{n, \text { odd } \geq 1} b_{n} \sin n \phi_{r}\right)$,
where $a_{n}=2\left\langle\cos n \phi_{r}\right\rangle$ and $b_{n}=2\left\langle\sin n \phi_{r}\right\rangle$. In this paper, $p$ ( $\phi_{r}$ ) is expanded to $n=24$, and it shows good convergence. The direction of $\mathbf{j}^{\prime}$ can be defined by the angle $\theta_{r}$ and $\phi_{r}$. In the CM frame, the distribution $p\left(\theta_{r}, \phi_{r}\right)$ can be expanded as
$p\left(\theta_{r}, \phi_{r}\right)=\frac{1}{4 \pi} \sum_{k q}[k] a_{q}^{k} C_{k q}\left(\theta_{r}, \phi_{r}\right)^{*}$

$$
\begin{equation*}
=\frac{1}{4 \pi} \sum_{k} \sum_{q \geq 0}\left[a_{q \pm}^{k} \cos q \phi_{r}-a_{q \mp}^{k} i \sin q \phi_{r}\right] C_{k q}\left(\theta_{r}, 0\right) \tag{5}
\end{equation*}
$$

During the calculation, the parameter $a_{q}^{k}$ is evaluated via
$a_{q \pm}^{k}=2\left\langle C_{k|q|}\left(\theta_{r}, 0\right) \cos q \phi_{r}\right\rangle_{\mathrm{k} \text { even }}$
$a_{q \pm}^{k}=2 i\left\langle C_{k|q|}\left(\theta_{r}, 0\right) \sin q \phi_{r}\right\rangle_{\mathrm{k} \text { odd }}$.
In this work, $p\left(\theta_{r}, \phi_{r}\right)$ is expanded up to $k=7$, which converged sufficiently.

## Quasi-classical trajectory calculations

In our QCT calculations, the KBNN potential-energy surface (PES) was employed. Kamisaka et al. modified the diatomics-in-molecules potential energy surface (DIM PES) and denoted the modified PES "KBNN PES" [8]. This consists of a $3 \times 3$ DIM potential matrix and three-body correction terms. Compared with the DIM PES, the KBNN PES has two distinguishing characteristics: first, the KBNN PES has a slightly shallower well in the lower PES; second, the long-range attractive force in the lower PES is smaller in KBNN. By diagonalizing the $3 \times 3 \mathrm{KBNN}$ potential matrix, the three adiabatic surfaces (the ground $1^{1} \mathrm{~A}^{\prime}$ surface, the first excited $2^{1} \mathrm{~A}^{\prime}$ surface, and the second excited $3^{1} \mathrm{~A}^{\prime}$ surface) can be obtained. The $3^{1} \mathrm{~A}^{\prime}$ surface is quite high, and the energy of the cross region between the $1^{1} \mathrm{~A}^{\prime}$ and $2{ }^{1} \mathrm{~A}^{\prime}$ surfaces is about 2.2 eV with respect to the asymptotic energy of the reagents. In our study, the collision energy selected was 0.7 eV , which is far below the cross region between the $1^{1} \mathrm{~A}^{\prime}$ and $2^{1} \mathrm{~A}^{\prime}$ surfaces. At such a low collision energy, nonadiabatic transitions [38, 39] will not take place. Thus, it is reasonable to perform the present calculation on the ground surface $\left(1^{1} \mathrm{~A}^{\prime}\right)$ with a potential well of $\sim 4.5 \mathrm{eV}$. Besides, the QCT method we
used in this work is the same as that used by Han et al., who first applied the QCT method to calculate product rotational alignment in 1993 [40], which made it feasible to directly compare theoretical calculations with experimental measurements of product polarizations.

## Results and discussion

Figure 2 shows the total reaction probability $P_{\mathrm{R}}\left(E_{\text {collision }}\right)$ as a function of the collision energy $E_{\text {collision }}$ for the $\mathrm{H}^{+}+\mathrm{D}_{2}$ $(v=0, j=0) \rightarrow \mathrm{D}^{+}+\mathrm{HD}$ reaction with the total angular momentum $J=0$. In the figure, the QCT results are compared with the weavepacket exact quantum mechanical (WP-EQM) ones (the black line) [2]. Despite the oscillations in the QM results over the investigated energy range, the QCTcalculated reaction probability agrees reasonably well with the WP-EQM result. Of course, there are some differences between the QCT and WP- EQM results, as indicated in [2]. However, based on the reasonable agreement shown above, we believe that such differences will not significantly influence the general trend in the stereodynamical properties obtained from the present QCT calculations.

The QCT PDDCSs of the $\mathrm{H}_{2}, \mathrm{HD}$ and $\mathrm{D}_{2}$ product molecules from the two reactions with two different channels are presented in Fig. 3. The PDDCS $[(2 \pi / \sigma)$ $\left(d \sigma_{00} / d \omega_{t}\right)$ ] describes the correlation of the $\mathbf{k}-\mathbf{k}^{\prime}$ or the scattering direction of the product. As can be seen in Fig.3a, for all product channels, $(2 \pi / \sigma)\left(d \sigma_{00} / d \omega_{t}\right)$ is roughly symmetric with respect to $90^{\circ}$, which demonstrates that a complex with a long lifetime (compared to the mean rotational period) forms during these reactions [2, 14, 20]. Although the values of $(2 \pi / \sigma)\left(d \sigma_{00} / d \omega_{t}\right)$ are quite small,


Fig. 2 Comparison of QCT and WP-EQM total reaction probabilities $P^{J=0+}\left(E_{\text {collision }}\right)$ as a function of the collision energy $E_{\text {collision }}$ with $J=0$, for the $\mathrm{H}^{+}+\mathrm{D}_{2}(v=0, j=0) \rightarrow \mathrm{D}^{+}+\mathrm{HD}$ reaction. Red dashed line QCT results. Black solid line WP-EQM results


Fig. 3a-d Four PDDCSs of the products at a collision energy of 0.7 eV . a-d show the PDDCSs with $(k, q \pm)=(0,0),(2,0),(2,2+),(2,1-)$, respectively. The reactions $\mathrm{H}^{+}+\mathrm{HD} \rightarrow \mathrm{H}_{2}+\mathrm{D}^{+}, \mathrm{H}^{+}+\mathrm{HD} \rightarrow \mathrm{HD}+\mathrm{H}^{+}$,
there are still some differences among these reactions. For the $\mathrm{H}^{+}+\mathrm{HD}$ reaction, the products of channel 1 show a slight backward bias, while the $(2 \pi / \sigma)\left(d \sigma_{00} / d \omega_{t}\right)$ values of the products of channel 2 are obviously symmetric with respect to $90^{\circ}$. For the $\mathrm{D}^{+}+\mathrm{HD}$ reaction, the products of channel 1 still slightly favor backward scattering, while the forward scattering of products is relatively strong in channel 2. In addition, in each reaction with different product channels, the distribution of the scattered products shows an increased tendency for backward scattering with an increasing mass factor $\cos ^{2} \beta$ [defined by $\cos ^{2} \beta=m_{\mathrm{L}} m_{\mathrm{L}^{\prime \prime}} /$ $\left(m_{\mathrm{L}}+m_{\mathrm{L}^{\prime}}\right)\left(m_{\mathrm{L}^{\prime}}+m_{\mathrm{L}^{\prime \prime}}\right)$ for the $\mathrm{L}+\mathrm{L}^{\prime} \mathrm{L}^{\prime \prime} \rightarrow \mathrm{LL}^{\prime}+\mathrm{L}^{\prime \prime}$ reaction $]$. In Table 1, we show the mass factors we calculated for these reactions. The distribution of $(2 \pi / \sigma)\left(d \sigma_{20} / d \omega_{t}\right)$, which is related to $\left\langle P_{2}\left(\cos \theta_{r}\right)\right\rangle$, is shown in Fig. 3b. Compared with the $\operatorname{PDDCS}(2 \pi / \sigma)\left(d \sigma_{00} / d \omega_{t}\right)$ values, the values of $(2 \pi / \sigma)\left(d \sigma_{20} / d \omega_{t}\right)$ are small for all angles between $0^{\circ}$ and $180^{\circ}$. It is very interesting to note that the
 b
d
$\mathrm{D}^{+}+\mathrm{HD} \rightarrow \mathrm{HD}+\mathrm{D}^{+}$, and $\mathrm{D}^{+}+\mathrm{HD} \rightarrow \mathrm{D}_{2}+\mathrm{H}^{+}$are depicted by a black solid line, a black dashed line, a red solid line and a red dashed line, respectively
distribution of $(2 \pi / \sigma)\left(d \sigma_{20} / d \omega_{t}\right)$ shows the opposite trend to that of the $(2 \pi / \sigma)\left(d \sigma_{00} / d \omega_{t}\right)$, which implies that the product's rotational angular momentum $\mathbf{j}$ ' is preferentially polarized along the direction perpendicular to $\mathbf{k}$ for these reactions. Comparing channel 1 with channel 2 , we can see that the values of $(2 \pi / \sigma)\left(d \sigma_{20} / d \omega_{t}\right)$ approach -0.5 in channel 2 of the two reactions. Therefore, we can say that, of these two channels, the products of channel 2 show

Table 1 Alignment parameters $\left\langle P_{2}\left(\mathbf{j}^{\prime} \cdot \mathbf{k}\right)\right\rangle$ calculated for $\mathrm{H}^{+}+\mathrm{HD}$ and its isotopic reaction

| Reaction system | $\left\langle P_{2}\left(\mathbf{j}^{\prime} \cdot \mathbf{k}\right)\right\rangle$ | Mass factor $\cos ^{2} \beta$ |
| :--- | :--- | :--- |
| $\mathrm{H}^{+}+\mathrm{HD} \rightarrow \mathrm{H}_{2}+\mathrm{D}^{+}$ | -0.04955 | 0.333 |
| $\mathrm{H}^{+}+\mathrm{HD} \rightarrow \mathrm{HD}+\mathrm{H}^{+}$ | -0.0302 | 0.111 |
| $\mathrm{D}^{+}+\mathrm{HD} \rightarrow \mathrm{HD}+\mathrm{D}^{+}$ | -0.02669 | 0.444 |
| $\mathrm{D}^{+}+\mathrm{HD} \rightarrow \mathrm{D}_{2}+\mathrm{H}^{+}$ | -0.07039 | 0.166 |

relatively strong alignment. The $\operatorname{PDDCSs}(2 \pi / \sigma)\left(d \sigma_{22+} /\right.$ $\left.d \omega_{t}\right)$ and $(2 \pi / \sigma)\left(d \sigma_{21-} / d \omega_{t}\right)$, which correlate with $\left\langle\sin ^{2} \theta_{r} \cos ^{2} \phi_{r}\right\rangle$ and $\left\langle\cos \theta_{r} \sin \theta_{r} \sin \phi_{r}\right\rangle$, are displayed in Fig. 3c and d, respectively. Compared with $(2 \pi / \sigma)\left(d \sigma_{00} /\right.$ $\left.d \omega_{t}\right)$, the values of $(2 \pi / \sigma)\left(d \sigma_{22+} / d \omega_{t}\right)$ and $(2 \pi / \sigma)\left(d \sigma_{21-} /\right.$ $d \omega_{t}$ ) are relatively small and are quite close to zero. This means that, on the whole, the product polarizations depicted by these two PDDCSs are very weak. Even so, Fig. 3c shows that the product $D_{2}$ displays relatively strong polarization at about $37^{\circ}$ and $142^{\circ}$ in channel 2 of the $\mathrm{D}^{+}+\mathrm{HD}$ reaction, while the polarization of the $\mathrm{H}_{2}$ product is relatively strong at $130^{\circ}$ in channel 1 . In Fig. 3d, the distribution of $(2 \pi / \sigma)\left(d \sigma_{21-} / d \omega_{t}\right)$ shows some oscillations in channel 1 of the $\mathrm{H}^{+}+\mathrm{HD}$ reaction. For the other reactions, the distribution of $(2 \pi / \sigma)\left(d \sigma_{21-} / d \omega_{t}\right)$ has only very small oscillations. Thus, the distribution of the product rotational angular momentum $\mathbf{j}^{\prime}$ tends to be anisotropic in channel 1 of the $\mathrm{H}^{+}+\mathrm{HD}$ reaction and in channel 2 of the $\mathrm{D}^{+}+\mathrm{HD}$ reaction.

Figures 4 and 5 show the distributions of $p\left(\theta_{r}\right)$ and $p\left(\phi_{r}\right)$ for these reactions with different product channels. As can be seen, the values of $p\left(\theta_{r}\right)$ and $p\left(\phi_{r}\right)$ are rather small, indicating that the polarization of the product is weak, which is caused by the long-lived complex formed in these reactions [14, 20]. In Fig. 4, the maximum for $p\left(\theta_{r}\right)$ is located at $90^{\circ}$, and the distribution is symmetric with respect to $90^{\circ}$ for these reactions. Figure 5 shows the distribution of $p\left(\phi_{r}\right)$, which describes the $\mathbf{k}-\mathbf{k}^{\prime}-\mathbf{j}^{\prime}$ correlation. Obviously, the values of $p\left(\phi_{r}\right)$ are small, suggesting that the rotational alignment of the product is not strong. However, the observation that $P\left(\phi_{r}\right)$ is asymmetric about $\phi_{r}=180^{\circ}$, indicates the existence of product orientation. The reason for this is probably the repulsive energy between the H and $D$ atoms. In [41-43], an impulse model for the $A+B C \rightarrow$


Fig. 4 The distributions of $p\left(\theta_{r}\right)$ for the product molecules at a collision energy of 0.7 eV . $p\left(\theta_{r}\right)$ describes the correlation $\mathbf{k}-\mathbf{j}^{\prime}$ for these reactions


Fig. 5 The distributions of $p\left(\varphi_{r}\right)$ for the product molecules at a collision energy of 0.7 eV . $p\left(\varphi_{r}\right)$ shows the correlation $\mathbf{k}-\mathbf{k}^{\prime}-\mathbf{j}^{\prime}$ for these reactions
$\mathrm{AB}+\mathrm{C}$ reaction was used to calculate the rotational alignment of the product molecules. Starting from this model [43], we have
$\mathbf{j}^{\prime}=\mathbf{L} \sin ^{2} \beta+\mathbf{J} \cos ^{2} \beta+\mathbf{J}_{\mathbf{1}} m_{B} / m_{B}$,
where $\mathbf{L}$ is the orbital angular momentum of the reagent, $\mathbf{J}$ is the rotational angular momentum of the reagent, and $\mathbf{J}_{\mathbf{1}}=\sqrt{\mu_{B C} R}\left(\mathbf{r}_{A B} \times \mathbf{r}_{B C}\right) ; \mu_{\mathrm{BC}}$ is the reduced mass of the BC molecule, $\mathbf{r}_{\mathrm{AB}}$ and $\mathbf{r}_{\mathrm{BC}}$ are unit vectors where B points to A and where B points to C , respectively, and $R$ is the repulsive energy. As we can see, in the above equation, $\mathbf{L} \sin ^{2} \beta+\mathbf{J} \cos ^{2} \beta$ is symmetric in these reactions, while the term $\mathbf{J}_{\mathbf{1}} m_{\mathrm{B}} / m_{\mathrm{AB}}$ associated with the repulsive energy $\mathbf{J}_{\mathbf{1}}$ shows a preferred direction. As a result of this preference, the products in our investigated reactions prefer right-handed rotation in the planes parallel to the scattering plane.

A comparison of the distributions of $p\left(\theta_{r}\right)$ for these reactions (see Fig. 4) also highlights some differences. For the $\mathrm{H}^{+}+\mathrm{HD}$ reaction, the distribution of $p\left(\theta_{r}\right)$ in channel 2 is broader than that in channel 1 , with a lower peak position at $\theta_{r}=90^{\circ}$ and the other two obvious and extra peaks at $0^{\circ}$ and $180^{\circ}$. Although both product channels exhibit a similar weak tendency for $\mathbf{j}^{\prime}$ to be aligned along the direction perpendicular to the reagent's relative velocity $\mathbf{k}$, it is obvious that the product's rotational alignment is stronger in channel 1. For the $\mathrm{D}^{+}+\mathrm{HD}$ reaction, the distribution of $P$ $\left(\theta_{r}\right)$ in channel 1 is broader than that in channel 2 , which demonstrates that in channel 1 the alignment of $\mathbf{j}^{\prime}$ is weaker than that in channel 2. This polarization behavior in $\mathrm{D}^{+}+\mathrm{HD}$ is quite different from that in the $\mathrm{H}^{+}+\mathrm{HD}$ reaction. When comparing the distribution of $P\left(\theta_{r}\right)$ in channel 1 of the $\mathrm{H}^{+}+\mathrm{HD}$ reaction with its corresponding part in the isotopic reaction $\mathrm{D}^{+}+\mathrm{HD}$, the oscillations in $P\left(\theta_{r}\right)$ are found to be less prominent in the $\mathrm{D}^{+}+\mathrm{HD}$ reaction, which shows that $\mathbf{j}^{\prime}$ is almost isotropic along the $\mathbf{k}$ direction in this
reaction system. Moreover, we calculated the alignment parameter $\left\langle P_{2}\left(\mathbf{j}^{\prime} \cdot \mathbf{k}\right)\right\rangle$, which can also reflect the degree of rotational polarization of the product. The values of $\left\langle P_{2}\left(\mathbf{j}^{\prime} \cdot \mathbf{k}\right)\right\rangle$ for these reactions are shown in Table 1, and are in good agreement with the distribution of $P\left(\theta_{r}\right)$.

For the $\mathrm{H}+\mathrm{H}^{\prime} \mathrm{L}(\mathrm{H}$, heavy; L, light) reaction, Han et al. [43, 44] found that the mass factor $\cos ^{2} \beta$ [defined as $\cos ^{2}$ $\beta=m_{\mathrm{L}} m_{\mathrm{L}^{\prime \prime}} /\left(m_{\mathrm{L}}+m_{\mathrm{L}}\right)\left(m_{\mathrm{L}^{\prime}}+m_{\mathrm{L}^{\prime \prime}}\right]$ for the reactions $\mathrm{L}+\mathrm{L}^{\prime} \mathrm{L}^{\prime \prime} \rightarrow$ $\left.\mathrm{LL}^{\prime}+\mathrm{L}^{\prime \prime}\right)$ can affect the distribution of the product's angular momentum vector. For the above reactions, the mass factor $\cos ^{2} \beta$ also has a significant impact on the product's rotational alignment. For the $\mathrm{D}^{+}+\mathrm{HD}$ reaction, the mass factor of channel $1\left(\mathrm{D}^{+}+\mathrm{HD} \rightarrow \mathrm{HD}+\mathrm{D}^{+}\right)$is larger than that of channel 2 (see Table 1). As a result, the calculated values for channel 1 become less negative and the distribution of $\mathbf{j}^{\prime}$ tends to become less anisotropic, while in channel 2 the rotation of the product is relatively strong aligned, as clearly shown in Figs. 3, 4, 5 and 6. A comparison of channel 1 for the $\mathrm{H}^{+}+\mathrm{HD}$ and $\mathrm{D}^{+}+\mathrm{HD}$ reactions shows the same phenomenon, which can be probably explained as follows. The total angular momentum is conserved during these reactions; that is, $\mathbf{j}+\mathbf{L}=\mathbf{j}^{\prime}+\mathbf{L}^{\prime}$ (here $\mathbf{L}$ and $\mathbf{L}^{\prime}$ are the reagent's and product's orbital angular momenta) [44]. Also, we have $\mathbf{j}^{\prime}=\mathbf{L} \sin ^{2} \beta+\mathbf{J} \cos ^{2} \beta+\mathbf{J}_{\mathbf{1}} m_{\mathrm{B}} / m_{\mathrm{AB}}$ [41]; under the condition that more angular momentum will be taken away by a heavier product ion, it is very likely that decreasing the mass factor will increase the anisotropy of the distribution of $\mathbf{j}^{\prime}$. In contrast, decreasing the mass factor will decrease the anisotropy of the distribution of $\mathbf{j}^{\prime}$ for the $\mathrm{H}^{+}+\mathrm{HD}$ reaction, and the same rule can be applied to channel 2 of both the $\mathrm{H}^{+}+\mathrm{HD} \rightarrow \mathrm{HD}+\mathrm{H}^{+}$and the $\mathrm{D}^{+}+\mathrm{HD} \rightarrow \mathrm{D}_{2}+\mathrm{H}^{+}$reactions. Aside from this, the lifetimes of the complexes and the threshold energies differ between the reactions, which is also responsible for the differences in behavior between these reactions.

From Fig. 5, the peaks for $p\left(\phi_{r}\right)$ appear at $90^{\circ}$ and $270^{\circ}$, which indicates that $\mathbf{j}^{\prime}$ is mainly aligned along the $y$ axis of the CM frame. For all reactions, the strong peak appears at $\varphi_{\mathrm{r}} \sim 90^{\circ}$, which demonstrates that the product molecules tend to show counterclockwise rotation. Upon comparing the two product channels in the $\mathrm{H}^{+}+\mathrm{HD}$ reaction, the peak is stronger in channel 1 than channel 2, which indicates that the product's rotational angular momentum in channel 1 is relatively closely oriented to the positive direction of the $y$ axis. It is also clear that the distribution of $P\left(\phi_{r}\right)$ in channel 2 is broader than that in channel 1. For the $\mathrm{D}^{+}+\mathrm{HD}$ reaction, the peak in channel 1 is weaker than that in channel 2, which indicates that the product's rotational angular momentum is relatively closely oriented to the

Fig. 6a-d Polar plots of $p\left(\theta_{r}, \varphi_{r}\right)$ distributions averaged over all scattering angles. a $\mathrm{H}^{+}+\mathrm{HD} \rightarrow \mathrm{H}_{2}+\mathrm{D}^{+}$reaction; $\mathbf{b} \mathrm{H}^{+}+\mathrm{HD} \rightarrow \mathrm{HD}+\mathrm{H}^{+}$ reaction; $\mathbf{c} \mathrm{D}^{+}+\mathrm{HD} \rightarrow \mathrm{HD}+\mathrm{D}^{+}$reaction; $\mathbf{d} \mathrm{D}^{+}+\mathrm{HD} \rightarrow \mathrm{D}_{2}+\mathrm{H}^{+}$ reaction




positive direction of the $y$ axis in channel 2. Upon comparing the same channel between the two different reactions, the orientation of the product molecules in channel 1 of the $\mathrm{H}^{+}+\mathrm{HD}$ reaction is more apparent than in its isotopic reaction $\mathrm{D}^{+}+\mathrm{HD}$. However, in channel 2, the orientations of the product molecules are similar for the two reaction systems. This indicates that either intramolecular or the intermolecular isotopic substitution can affect the product polarizations of the reaction systems.

Figure 6 shows the angular momentum polarization in the form of polar plots $p\left(\theta_{r}, \phi_{r}\right)$, which represents the full average distribution of $P\left(\omega_{t}, \omega_{r}\right)$ of the scattering angle. We can see that the distribution of $p\left(\phi_{r}, \phi_{r}\right)$ mainly peaks at $(\pi / 2$, $\pi / 2)$ and $(\pi / 2,3 \pi / 2)$, and this is in good agreement with the distributions of $P\left(\theta_{r}\right)$ and $P\left(\phi_{r}\right)$. These distributions of $p$ $\left(\theta_{\mathrm{r}}, \phi_{\mathrm{r}}\right)$ indicate that the $\mathrm{H}_{2}, \mathrm{HD}$, and $\mathrm{D}_{2}$ products are preferentially polarized perpendicular to the scattering plane, and these reactions are dominated by "in-plane" mechanisms.

Finally, we note that the overall product alignment shown in the present work is rather weak. This is especially apparent in Fig. 4, where the alignment at intermediate collision angles is rather low, and in Fig. 3c-d, where the calculated values are close to zero. Very interestingly, in a previous quantum study by Zanchet et al. [45], a similar finding was reported for the $\mathrm{H}^{+}+\mathrm{D}_{2}$ reaction. Their quantum study showed that the calculated alignment values of the product HD are nearly always close to zero at almost all scattering angles for the $\mathrm{H}^{+}+\mathrm{D}_{2}(v, j) \rightarrow \mathrm{HD}\left(v^{\prime}, j^{\prime}\right)+\mathrm{D}^{+}$ reaction at a translational energy of 0.524 eV , thus clearly indicating that $\mathbf{j}^{\prime}$ and $\mathbf{k}^{\prime}$ are randomly aligned. Thus, our present stereodynamics study based on QCT calculations has further corroborated this argument from the previous quantum study.

## Conclusions

The quasi-classical trajectory method has been employed to investigate the isotopic effects on the reactions $\mathrm{H}^{+}+\mathrm{HD}$ and $\mathrm{D}^{+}+\mathrm{HD}$ using the ground KBNN PES at a collision energy of 0.7 eV . The polarization-dependent differential crosssection and the distributions of $p\left(\theta_{r}\right)$ and $p\left(\phi_{r}\right)$ have been calculated. Comparison of the total reaction probability with $J=0$ for the $\mathrm{H}^{+}+\mathrm{D}_{2}$ reaction shows good agreement with the wavepacket exact quantum-mechanical result. The distributions of $P\left(\phi_{r}\right)$ right-handed product rotation in planes parallel to the scattering plane for these reactions. It was also found that the mass factor plays an important role in product rotation, with different effects on product's rotational alignment for different reaction types. For the $\mathrm{D}^{+}+\mathrm{HD}$ reaction, the mass factor increases from channel 2 to channel 1 , and the distribution of the product's rotational
angular momentum becomes less anisotropic, while increasing the mass factor enhances the anisotropy of the distribution of $\mathbf{j}^{\prime}$ for the $\mathrm{H}^{+}+\mathrm{HD}$ reaction. For channel 1 in both reactions, $\mathrm{H}^{+}+\mathrm{HD}$ and $\mathrm{D}^{+}+\mathrm{HD}$, the isotropic distribution of $\mathbf{j}^{\prime}$ increased with increasing mass factor, but in channel 2 the isotropic distribution of $\mathbf{j}^{\prime}$ decreased with increasing mass factor. Aside from the mass factor, other factors, such as the lifetime of the complex and the threshold energy, can also influence the polarizations of the products for these reactions.

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